Networked Predictive Control Systems Based on the Hammerstein Model

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Abstract—In this paper, a novel predictive control-based approach is proposed for a networked control system with random delays containing an input nonlinear process based on a Hammerstein model. The method uses a time-delay two-step generalized predictive control scheme, which consists of two parts: one is to deal with the input nonlinearity of the Hammerstein model and the other is to compensate for the network-induced delay in the networked control system. A theoretical result using the Popov criterion is presented for the closed-loop stability of the system in the case of a constant delay. Simulation examples illustrating the validity of the approach are also presented.

Index Terms—Hammerstein model, networked control systems, predictive control, time-delay compensator, two-step approach.

I. INTRODUCTION

A CONTROL system is called a “networked control system” (NCS) when the link from sensor to controller and/or the link from controller to actuator are/is connected not directly as is assumed in conventional control systems but via a serial communication network with limited resources [1], [2]. This configuration, due to the advantages the network introduces, brings to the system lower cost, flexibility, and the ability of remote control while at the same time greatly degrades the performance of the control system or even makes the system unstable under certain conditions, due to the disadvantages the network introduces, such as the time delay (so called “network-induced delay”), data packet dropout, and quantization. Such an implementation presents a new challenge to conventional control theory.

A large number of papers have addressed NCSs to date, considering different issues, mainly on the treatment of the network-induced delay. The majority of papers published have largely been restricted to linear systems [3], [4]. In this paper, we will consider a particular category of nonlinear system represented by a Hammerstein model [5], which consists of a cascade connection of a static nonlinearity followed by a dynamic linear time-invariant (LTI) system. This nonlinear model is important in theory and applies to a number of practical applications, see, e.g., [6] and [7]. To deal with the control problem when such a nonlinear system is implemented in a networked control environment, a time-delay two-step generalized predictive control (TDTSGPC) approach is proposed in this paper. In this approach, the nonlinear input process is dealt with using the two-step design approach developed in [8], and a predictive-based compensation scheme is also designed to compensate for the network-induced delay. It works as follows. The predictive control method is first applied to the linear part of the Hammerstein model to generate the intermediate control predictions. It is assumed that, for the nonlinear part, numerical methods can be used to obtain the real control predictions from the intermediate control predictions, and then a sequence of the predictions is packed and sent to the actuator through the network simultaneously. At every execution time, the specially designed time-delay compensator will select the appropriate control signal to compensate for the network-induced delay. Using this compensation scheme, the network-induced delay can be exactly compensated for in an active way. The stability criterion of the proposed TDTSGPC approach is obtained using the Popov theorem. Simulations are also done to illustrate the validity of the approach.

The remainder of this paper is organized as follows. The design of TDTSGPC based on a Hammerstein model is presented in Section II. Then, the theoretical results for the system stability and the simulation results of TDTSGPC are presented in Sections III and IV, respectively. The paper gives the conclusions in Section V.

II. DESIGN OF TDTSGPC BASED ON THE HAMMERSTEIN MODEL

The following Hammerstein system $S$ is considered in this paper, with the combination of the CARIMA model and a static input nonlinear function $f(\cdot)$ as

$$S : \left\{ \begin{array}{l}
   \alpha y(k) = b u(k-1) + \xi(k)/\Delta \\
   y(k) = f(u(k))
\end{array} \right. \quad (1a) \quad (1b)$$

where $\xi(k)$ is the Gaussian white noise with zero mean value, $u(k), y(k), f(y(k))$ is the input, intermediate input, and output at time $k$, respectively. $\Delta = 1 - z^{-1}, a = 1 + a_1 z^{-1} + \cdots + a_n z^{-n}, b = b_0 + b_1 z^{-1} + \cdots + b_m z^{-m}$ with $a_n \neq 0, b_m \neq 0$, and the input nonlinear function $f(\cdot)$ is memoryless, static with $f(0) = 0$.

The network-induced delay is one of the key problems when a control system is implemented in a networked control environment [1], which will make the current control input unavailable to the actuator. It is also an important problem in conventional time-delay systems (TDSs), in which there are mainly two ways to deal with this situation. This is to use either the last available control signal or to use zero control [9]. In both methods, the previous information of the system, including the system states, outputs and inputs, and the structure information of the system is not considered. However, with the use of the network in NCSs, it is possible to send a sequence of the control signals together due to the packet-based transmission of the network. Thus, in order to compensate for the network-induced delay, a sequence of forward control predictions can be calculated and
sent to the actuator simultaneously, from which the appropriate control input can be picked out to actively compensate for the network-induced delay [12]. Since more information is used in this compensation approach, a better performance can be expected than that obtained in conventional TDSs. Following this idea, in this paper, the Linear Generalized Predictive Control (LGPC) approach is applied to generate the control predictions of the linear part of the Hammerstein model while the nonlinear part remains to be solved using a numerical method. This is why it is called a “two-step” approach for a Hammerstein model [8].

The two parts of TDTSGPC, the design of the two-step generalized predictive control (TSGPC) and the time-delay compensator, will now be considered.

A. Design of TSGPC

The key idea of TSGPC is to first design the intermediate control sequence \( v(k) \) of the linear part of Hammerstein model (1a) with the LGPC method and then obtain the real control sequence \( u(k) \) from the relationship \( u(k) = f(v(k)), k = 1, 2, \ldots, N_u \), where \( N_u \) is the control horizon [8]. Hence, we will first present the design of LGPC and then describe how to obtain the real inputs from the nonlinear relationship in the following subsections.

1) Design of LGPC: Without consideration of the input nonlinearity of the Hammerstein model, the LGPC problem for (1a) is solved for the following objective function:

\[
\min J(k) = ||Y(k) - \varpi||^2_Q + ||\Delta V(k)||^2_R
\]

where \( \varpi = [\omega_1, \omega_2, \ldots, \omega_{N_u+N_f+N_v}] \) is the set-point, weight matrices \( Q_{N_u+N_f+N_v} \) are diagonal, \( Y(k) = [y(k), y(k+1), \ldots, y(k+N_y)]^T \), \( y(k+i), i > 0 \), are the outputs, \( \Delta V(k) = \Delta v(k), \Delta v(k) = [\Delta v(k), \Delta v(k+1), \ldots, \Delta v(k+N_y-1)]^T \), \( N_f \) is the delay of the feedback channel, \( N_p \) is the predictive horizon, and \( ||.||^2_Q \) means \( y^T Q y \).

In [12], the previous control sequence \( v(k-1), \ldots, v(k-N_f) \) is used to generate the control predictions at the controller side at time \( k \), while in reality this information is hard to obtain for the controller due to the time delay in both channels. In this paper, we propose a new method to deal with this problem, in which only the control and output information before time \( k - N_f \) are used to calculate the predictive control sequence by including the control sequence from time \( k - N_f \) to \( k - 1 \) as part of the predictive control sequence. This is obtained using the objective function above and the equation of the prediction controls below.

Introduce the following Diophantine equations:

\[
\begin{align*}
1 &= E_{F1} \Delta + z^{-j_{\tau_{ec}}} F_{j} \\
L &= j_{\tau_{ec}} + 1, \ldots, N_p, \quad \text{when } m > 0
\end{align*}
\]

where \( F_j = 1 + e_{j_1} z^{-1} + \cdots + e_{j_{\tau_{ec}}} z^{-j_{\tau_{ec}}-1}, F_j = f_{j_0} + f_{j_1} z^{-1} + \cdots + f_{j_{\tau_{ec}}} z^{-j_{\tau_{ec}}}, E_{S} = E_{Fj} + e_{j_1} z^{-1} + \cdots + e_{j_{\tau_{ec}}} z^{-j_{\tau_{ec}}-1}, e_{j_1} z^{-1} + \cdots + e_{j_{\tau_{ec}}} z^{-j_{\tau_{ec}}-1}, E_{S} = E_{Fj} + e_{j_1} z^{-1} + \cdots + e_{j_{\tau_{ec}}} z^{-j_{\tau_{ec}}-1}, \)

Define \( E = [E_{S1}, E_{S2}, \ldots, E_{S_{N_p}}]^T \), if \( m > 0; 0_{N(N_u+N_f+N_v)_x1} \), otherwise; \( G \in R^{N_u+N_f+N_v} \times (N_u+N_f+N_v) \),

with all the entries 0 but \( G(k,j) = 1 \) if \( m > 0; G(k,j) = 0 \), otherwise, for \( j = j_{\tau_{ec}} + 1, j_{\tau_{ec}} + 2, \ldots, N_p \);
\[ f(\cdot) \text{ by } \hat{f}^{-1}(\cdot) \text{ and assume that this nonlinearity due to the inaccuracy of calculation satisfies for some } K > 0 \]
\[ 0 \leq f \cdot \hat{f}^{-1}(\theta) \leq K\theta. \] (8)

From the discussion above, the real predictive control sequence of TSGPC can be obtained as
\[ U(k|k - \tau_{sc}) = \hat{f}^{-1}(V(k|k - \tau_{sc})) \] (9)

where
\[
\hat{f}^{-1}(\cdot) = \left( \hat{f}^{-1}(\cdot) \hat{f}^{-1}(\cdot) \cdots \hat{f}^{-1}(\cdot) \right)^{1/N_{uc}+1} U(k|k - \tau_{sc}) = \left( u(k|k - \tau_{sc}) \cdots u(k+N_{uc}-1|k - \tau_{sc}) \right)^T.
\]

Remark 2: It is necessary to point out that what is required in implementing TSGPC is to satisfactorily meet the sector constraint in (8), no matter how the inverse function \( \hat{f}^{-1}(\cdot) \) is calculated. It implies that the function \( f(\cdot) \) does not need to be invertible as long as its inverse can be obtained by a numerical method and satisfies the sector constraint. One can refer to [11] and the references therein for more information of the calculation of \( \hat{f}^{-1}(\cdot) \).

B. Design of Time-Delay Compensator

The network introduces to the NCSs not only delays but also an advantage to the system in that a sequence of signals can be packed and transmitted simultaneously [12], [13]. Our time-delay compensator takes advantage of this characteristic of NCSs.

The following assumptions are first made in the time delay compensator design.

1) For the sake of the calculation of the predictive control sequence, the time delay of the feedback channel needs to be known to the controller, which can be easily done by issuing a time stamp on each data packet from the sensor side to the controller side.

2) The round-trip time (RTT), noted by \( \tau \), the total time delay of feedback channel and forward channel, i.e., \( \tau = \tau_{sc} + \tau_{ca} \) is known to the actuator, which can also be done by using the time stamps.

3) The predictive control sequences are packed and transmitted to the actuator simultaneously.

4) The forward time delay is less than the control horizon \( N_{uc} \). The time-delay compensator works as follows: at every time instant \( k \), the predictive controller calculates a sequence of future control signals based on the outputs and control information before \( k - \tau_{ca,k} \) (the time stamp of the packet received at time \( k \) of the controller side). The future control signals are then transmitted to the actuator side with a time stamp \( k \) all in one packet. When a packet of a control sequence arrives at the actuator side (different packets may experience different time delays), it is compared with the one already in the cache of the actuator according to the time stamp, and only the latest is saved. The actuator then chooses the control action \( u(k + \tau_{ca,k}|k - \tau_{sc,k}) \) if the time stamp of the control sequence in its cache is \( k \) and the forward time delay is \( \tau_{ca,k} \).

Using this compensation scheme, the network-induced delays can be exactly compensated for.

III. STABILITY ANALYSIS

Here, we first give the explicit expression of the closed-loop system using the two-step predictive control approach and the delay compensator and then obtain the stability criterion of the closed-loop system using a Popov criterion.

A. Closed-Loop System

Note that the time instant \( k \) in the time compensator described above is based on the time at the controller side. Let \( \tau_{ca,k}^* \) denote the time delay in the forward channel of the control sequence which is applied by the actuator at time instant \( k \) (the time at the plant side), and then the time stamp of this sequence (the time when it is sent at the controller side) is
\[ k^* = k - \tau_{ca,k}^* = \max_j\{jU(j|j - \tau_{ca,j}) \in \Gamma_k\} \] (10)
where \( \tau_{ca,j} \) is the delay in the feedback channel corresponding to time instant \( j \) at the controller side, and \( \Gamma_k \) is the set of control sequences that are available at time interval \( (k - 1, k) \) at the actuator side, which includes the one in the cache of the actuator and any one that arrives at the actuator between this interval.

From (9) and (10), the control signal adopted by the actuator at time \( k \) is obtained as
\[ u(k) = d_{ca,k}^c U(k - \tau_{ca,k}^*|k - \tau_{k}^*) \] (11)

where \( d_{ca,k}^c \) is a \( N_{uc} \times 1 \) column vector with all entries 0 but the \( \tau_{ca,k}^* \)th is 1, \( \tau_{k}^* \) is the RTT with respect to \( \tau_{ca,k}^* \), i.e., \( \tau_{k}^* = \tau_{ca,k}^* + \tau_{ca,k}^{2} \).

Combining (1a), (5), (6), (7), (9), and (11), the TDTSGPC approach applied to a Hammerstein model can be fully described by the following equations (\( \omega \) is set to 0 without loss of generality):
\[
\begin{align*}
    a_p(k) &= b(k - 1) \quad (12) \\
    v(k) &= f(u(k)) \quad (13) \\
    u(k) &= d_{ca,k}^c \hat{f}^{-1}(V(k - \tau_{ca,k}^*|k - \tau_{k}^*)) \quad (14) \\
    V(k - \tau_{ca,k}^*|k - \tau_{k}^*) &= PMF(k - \tau_{k}^* - 1) - PCDE \cdot \Delta v(k - \tau_{k}^* - 1) - PCDFy(k - \tau_{k}^*), \quad (15)
\end{align*}
\]

From the definition of \( d_{ca,k}^c \), \( v(k) \), and \( V(k - \tau_{ca,k}^*|k - \tau_{k}^*) \), we obtain
\[
    v(k - \tau_{k}^* - 1) = z^{-\eta_{k}^{(1)}} d_{ca,k}^c V(k - \tau_{ca,k}^*|k - \tau_{k}^*),
\]

Combining with (15), we then obtain
\[
    V(k - \tau_{ca,k}^*|k - \tau_{k}^*) = \left( z^{-\eta_{k}^{(1)}} PMF d_{ca,k}^c \right. \left. - z^{-\eta_{k}^{(1)}} PCDE \Delta d_{ca,k}^c - I \right)^{-1} \times PCDFy(k - \tau_{k}^*) \quad \text{at} \quad \tau_{k}^*,
\]

where the Popov criterion is applied to prove the stability of the TDTSGPC approach for constant delays.

Note that the choice of the predictive control signal to be applied to the plant depends on the network-induced delay in the forward channel. This is different from conventional GPC applications where the first control signal is always used.
Lemma 1 (Popov Criterion, See [8]): Suppose that $H(z^{-1})$ in Fig. 1 is stable and $0 \leq \Phi(\theta) \leq K\theta$. Then, the closed-loop system is stable if $1/K + iK + H(z^{-1}) > 0, \forall |z| = 1$.

In the case of constant delays, we have that $\tau^* = \tau^*_{\text{crack}} = \tau^*_{\text{net}} \forall k$, are all constant. Applying Lemma 1 to TDTSGPC and denote the characteristic polynomial of a transfer function $H(z^{-1})$ by $\delta(H(z^{-1}))$, we then obtain the following theorem.

Theorem 1: Suppose that the linear part of the Hammerstein model is accurate and the roots of $\delta(A_r(z^{-1})) = 0$ are located in the unit circle. Then, the closed-loop system of TDTSGPC is stable if there exists a positive constant $K$ such that the following is satisfied.

1) The input nonlinearity of the plant satisfies

$$0 \leq v \leq K\overline{v},$$

(17)

2) The network induced delay satisfies

$$\frac{1}{K} + iK + A_r(z^{-1}) > 0, \forall |z| = 1$$

(18)

where $A_r(z^{-1}) = (z^{-\tau^*}_{\text{net}} - 1) \overline{\mathcal{F}}(z^{-1}) L_r(z^{-1})b)/(a)$, and $\overline{v}(k) = A_r(z^{-1})v(k)$ is the theoretical input value to the CARIMA model.

Proof: Without loss of generality, assume $\omega = 0$. Notice here that, for any column vector $P$ with comparable dimensions,

$$f(d_{\text{col}}^* f^{-1}(P)) = f \cdot f^{-1}(d_{\text{col}}^* P)$$

by the definition of $f^{-1}(\cdot)$. Then, from (12)–(16), we obtain

$$v(k) = f(u(k))$$

$$= f(d_{\text{col}}^* f^{-1}(V(k - \tau^*_{\text{crack}})(k - \tau^*))$$

$$= f \cdot f^{-1}(d_{\text{col}}^* L_r(z^{-1})v(k))$$

$$= f \cdot f^{-1}(A_r(z^{-1})v(k))$$

$$= f \cdot f^{-1}(\overline{v}(k)).$$

(19)

This is equivalent to the block diagram shown in Fig. 2. Thus, the theorem can be easily obtained by applying Lemma 1 to Fig. 2.

IV. SIMULATION

We give an example to illustrate the TDTSGPC approach in this section. The linear part of the system adopted is $y(k) - 0.8y(k-1) = 2u(k-1)+3v(k-2)$, and the input nonlinearity of the Hammerstein model is chosen as $v = f(u) = u^2$ and the practical inverse of $f(\cdot)$ is $f^{-1}(\cdot) = \sqrt{u} \times \epsilon$, where $\epsilon$ is a random number with a uniform distribution in $[0,1]$. This is introduced to represent the uncertainty in a practical implementation. From condition (1) of Theorem 1, we see that the parameter $K$ is 1 and the predictive horizon and control horizon are chosen as $N_p = N_d = 12$. It can be shown that the system is stable only for the first two cases according to Theorem 1 since for too large a time delay the system will not satisfy condition (2) in Theorem 1. The simulation results of three cases: 1) $(\tau_{\text{crack}}^*, \tau_{\text{net}}^*) = (0,0)$; 2) $(\tau_{\text{crack}}^*, \tau_{\text{net}}^*) = (2,3)$; and 3) $(\tau_{\text{crack}}^*, \tau_{\text{net}}^*) = (3,7)$ are shown in Figs. 3 and 4 to illustrate the validity of the theoretical analysis.

V. CONCLUSION

In this paper, the two-step generalized predictive control approach, which is usually used in the controller design for the Hammerstein model, is integrated with a time-delay compensator to deal with networked control systems based on a Hammerstein model with random network-induced delays. This approach takes advantage of the characteristic of the network in an NCS such that a sequence of information can be packed
to be transmitted simultaneously, so that the predictive control method can be easily implemented for NCSs. A theoretical result is presented for the stability of the system in the case of a constant time delay. Simulation work has also been done to illustrate the validity of the approach. Further research is still needed to analyze the stability conditions under random time delays, which is not addressed in this paper.

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