Stability of a class of Markovian jumping nonlinear systems with time-delay in detection of switching signal

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Abstract: The asynchronous stability of a class of Markovian jumping nonlinear systems with time-delay in detection of switching signal is investigated. In the model, the detection delay is modeled Markovian and dependent on the actual Markovian jump signal. By constructing the constraint relationships between instability margin or detection delay and the mode transition rate of the underlying Markov process, the sufficient criterion for \( p \)th moment exponentially stable is obtained. It is shown that the stability of the considered system can be guaranteed by a sufficiently small mode transition rate. Finally, a numerical example illustrates the effectiveness of the obtained results.

Keywords: Nonlinear systems, Markovian parameter, asynchronous switching, \( p \)th moment exponentially stability.

1. INTRODUCTION

The study of switched systems is a topic of increasing interest [1]–[4]. Informally, switched systems is a family of continuous-time (or discrete-time) dynamical subsystems and a switching rule that governs the switching between them, which are used to model physical or man-made systems displaying switching features due to jumping parameters or changing environmental factors. Specifically, it is called a randomly switched system if the mode switches are governed by a stochastic process statistically independent from the system states. Furthermore, a Markovian jump linear system (MJLS) if the stochastic process is Markovian and the system dynamic is linear [5], which has received considerable attentions in theory [6]–[10].

The switches in switched systems are often assumed to be strictly synchronized [1], [2], which may not generally hold in reality due to unknown and unpredictable issues such as time-delay, disturbance, component and interconnection failures, etc. Thus the so-called asynchronous switching is proposed, and a number of efforts have been made in this area, for example, state

feedback stabilization [13], input-to-state stabilization [14], and output feedback stabilization [15], the use of the average dwell time approach [16]–[19], discrete-time MJLSs [20], [21], just to name a few. For the case of continuous-time MJLS, Mariton [5] and Kang [22] did the research, in which the stability of the continuous-time MJLSs with detection delays and false alarms in detected switching signal are considered, and two Markov processes conditional on the real Markovian switching signal are proposed to modeled the processes of detection delay and false alarm. However, almost all the research on asynchronous randomly switched linear systems, while the asynchronous randomly switched nonlinear systems have received little attention, which motivates this study for us.

In this paper, we will consider the asynchronous stability of a class of nonlinear systems with Markovian jump parameters. Compared with the asynchronous deterministic switched systems, the stability analysis for asynchronous Markovian jumping nonlinear systems is different from the above work. On the one hand, since the considerable system is a random one, the dwell time approach and average dwell time approach in deterministic switched systems can not be used to deal with asynchronous stability analysis of such systems. In addition, the detected switching signal is still a Markov process, and further increases the complexity of the problem. On the other hand, we consider the general nonlinear systems, its treatment method is different from the one for MJLSs with detection delay and false alarm via LMI method. All those increase the difficulty of our work. In the sequel, we will consider the asynchronous case with random detection delay and model the detected switching signal as a Markov process conditional on the real Markovian switching signal. The sufficient criteria for \( p \)th moment exponentially stability are given. It is shown that, the stability of Markovian jumping nonlinear systems with random detection delay in detected switching signal can be guaranteed provided that the mode transition rate is sufficiently small, which is similar

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to the conclusion in asynchronous deterministic switched systems based on average dwell time approach, i.e., the closed-loop conclusion can be guaranteed by a sufficient large average dwell time.

The remainder of the paper is organized as follows. The problem is formulated and necessary definitions and lemmas are given in Section II. The $p$th moment exponentially stability is then discussed in Section III. Then, the main results will be extended to a special class of Markov jumping nonlinear systems, and a numerical example with simulation results is given in Section IV. Section V concludes the paper.

**Notions:** $\mathbb{X}_+$ and $i_+$ denote the set of positive integer and nonnegative real numbers, respectively. $i^n$ and $i^{n \times m}$ denote $n$-dimensional real space and $n \times m$ dimensional real matrix space, respectively. For vector $x \in i^n$ , $|x|$ denotes the Euclidean norm . The transpose of vectors and matrices is denoted by superscript $T$. $C^{l,k}$ denotes all the functions with $l$th continuously differentiable first component and $k$th continuously differentiable second component.

### 2. Preliminaries

Consider the following asynchronous Markovian switching nonlinear systems

$$\begin{cases}
\mathcal{A}(t) = f(t,x(t),u(t),r(t)) \\
u(t) = h(t,x(t),r'(t))
\end{cases}$$

(1)

with the initial state $x_0$ and $r_0 = r(0) = i_0$ , where $x(t) \in i^n$ and $u(t) \in i^m$ are the state and asynchronous control input vectors, respectively. $r(t)$ is a right-continuous Markov process on the probability space taking values in a finite state space $S = \{1,2,L,N\}$ with transition probability matrix $P(t) = \{p_{ij}(t)\}_{N \times N}$ given by

$$p_{ij}(\Delta) = P\{r(t+\Delta) = j \mid r(t) = i\} = \begin{cases}
\pi_{ij}\Delta + o(\Delta), & i \neq j \\
1 + \pi_{ii}\Delta + o(\Delta), & i = j
\end{cases}$$

(2)

where $\Delta > 0$. $\pi_{ij} \geq 0$ is the transition rate from $i$ to $j$ ($i \neq j$), and $\pi_{ii} = - \sum_{j \neq i} \pi_{ij}$ is the detected signal of $r(t)$. Let $\bar{\pi} \triangleq \max_{i,j \in S} \{\pi_{ij}\}$, $\bar{\pi}' \triangleq \max_{i,j \in S} \{\pi_{ij}\}$ and $\Pi = \{\pi_{ij}\}_{N \times N}$. $d(t)$ is the detection delay. Moreover, $f : i^n \times n \times S \times i^m \rightarrow i^n$ and $h : i^n \times n \times S \rightarrow i^m$ are measurable functions with $f(t,0,0,i) \equiv 0$ and $h(t,0,0,i) \equiv 0$ for any $i \in S$. Let

$$\mathcal{F}(t,x(t),r(t)) = f(t,x(t),h(t,x(t),r'(t)),r(t))$$

(3)

For convenience, let $\mathcal{F}_i(t,x(t),u(t))$ denote $\mathcal{F}(t,x(t),u(t),i,j)$, for any $i, j \in S$. Specifically, it is said that the switches between the mode-dependent controller and the system are synchronous if $i = j$, and the switches are asynchronous if $i \neq j$. It is also assumed that $\tilde{f}$ satisfies the local Lipschitz condition and the linear growth condition. The closed-loop system can be rewritten as

$$\mathcal{A}(t) = f(t,x(t),r(t),r'(t))$$

(4)

and there exists an unique solution on $t \geq 0$.

In what follows $r(t)$ is assumed to be a regular Markov process with standard transition probability matrix. Let the sequence $\{t_l\}_{l \geq 0}$ denotes the switching instants sequence of $r(t)$, and $r(t^+_l) = i_l$, $t_0 = 0$. When $i_l = i$, $t_{l+1} - t_l$ is called the sojourn-time of Markov process in mode $i$. As usual, the sojourn-time sequence $\{t_{l+1} - t_l\}_{l \geq 0}$ belongs to an exponential distribution with rate parameter $\lambda(i)$, where $0 \leq \lambda(i) < \infty$ is the transition rate of $r(t)$ in mode $i$. In addition, we consider the detection signal with $r'(t) = r(t-d(t))$, and $\{t_r\}_{r \geq 0}$ denotes the switching instants sequence of $r'(t)$. As in [5], the following statements are assumed to describe the characteristic of $r'(t)$. When $r(t)$ has jumped from $i$ to $j$, $r'(t)$ follows with a delay that is also an independent exponentially distributed random variable with the mean $\frac{1}{\pi_{ij}}$ and

$$\mathbb{P}\{r'(s) = i, s \in [r^+,t]\} = \begin{cases}
\pi_{0i}^0\Delta + o(\Delta), & i \neq j \\
1 + \pi_{ii}^0\Delta + o(\Delta), & i = j
\end{cases}$$

(5)

Clearly, when letting $\pi_{ij}^0 \rightarrow \infty$, the detection is instantaneous. It is assumed that $\pi_{ij}^0$ is sufficiently large and $0 \leq d(t) \leq d = \inf\{t_{l+1} - t_l\}$. And further, $r'(t)$ is causal, which means that the ordering of the switching instants of $r'(t)$ is the same as the ordering of the corresponding switching instants of $r(t)$. Thus, it follows $0 = t_0 = t_{i'_l} < t_1 \leq t_{i'_f} < t_2 \leq t_{i'_g} < L < t_{l_f} \leq t_{i_{l+1}} < L$ , where $t_{i'_l} = t_l + d(t_l)$ for any $l \geq 1$.

**Remark 2.1:** In [5], both the detection delay and false
alarms are considered. Here, for simplicity, we don't consider the false alarms, which will be considered in future work.

Define a virtual switching signal $\tau(t)$, from $[0, \infty)$ to $S \times S$, by $\tau(t) = (r(t), r'(t))$. Let $\{\tau_l\}_{l \geq 0}$ denote the switching instants of $\tau(t)$. Then, for any $l \geq 1$, $\tau_0 = t_0$, $\tau_{l+1} = t_l$ and $\tau_l = t_r$. The following definitions and lemma are needed for the stability of system (3).

**Definition 2.1:** [23] The solution of system (3) is said to be $p$th moment exponentially stable, if for all $x_0 \in \mathbb{R}^n$ and $t_0 \in S$, there exist $M > 0$ and $\lambda > 0$, such that

$$E[|x(t)|^p] \leq ME[|x_0|^p]e^{-\lambda(t-t_0)}, \forall t \geq t_0$$  \hspace{1cm} (5)

**Definition 2.2:** [5] For any given $V(x(t), t, \tau(t)) = V(x(t), r(t), r'(t))$ in $C_{++}^2(\mathbb{R}^n \times _{++} \times S \times S)$, associated with system (3), the infinitesimal generator $L$, from $\mathbb{R}^n \times _{++} \times S \times S$ to $i$, can be described as follows

**Case 1.** When $r'(t) = r(t) = i$, then

$$L V(x(t), t, i, i) = V_i(x(t), t, i, i)$$

**Case 2.** When $r'(t) = i, \ r(t) = j$ and $j \neq i$, then

$$L V(x(t), t, j, i) = V_j(x(t), t, j, i) - \pi_{ji}^0 V(x(t), t, j, j) - \pi_{ij}^0 V(x(t), t, i, i)$$

**Lemma 2.1:** [22] Suppose that $r(t)$ is a Markov process. Then there exists $M > 0$ such that for all $t \geq 0$, the moment generating function $E[e^{N_r(t, 0)}]$ of $N_r(t, 0)$ satisfies

$$E[e^{N_r(t, 0)}] \leq M + e^{\rho \tau - \Phi \lambda} \forall \tau \geq 0$$  \hspace{1cm} (8)

where $N_r(t, 0)$ denotes the number of switches of $r(t)$ on the time-interval $[0, t]$.

**3. pTH MOMENT EXPONENTIALLY STABILITY**

In this section, the $p$th moment exponentially stability will be considered, and the corresponding sufficient criterion is obtained. To begin with the main results, an useful lemma is obtained.

**Lemma 3.1:** Let $V(t) = e^{\lambda t}V(x(t), t, \tau(t)) = e^{\lambda t}V(x(t), t, r(t), r'(t))$ for all $t \geq 0$ and $\lambda > 0$, then

$$D^T E\{V(t)\} = E\{L V(t)\}$$

$$= \lambda E\{V(t)\} + e^{\lambda t}E\{\{V(x(t), t, r(t), r'(t))\}\}$$  \hspace{1cm} (9)

where $D^T E\{V(t)\} = \limsup_{dt \to 0^+} \frac{E\{V(t + dt)\} - E\{V(t)\}}{dt}$.

**Proof:** Firstly, for any $k_1, k_2 \in S$, it follows

$$E\{V(t + dt)\} | x(t), r(t) = k_1, r'(t) = k_2, t\} = E\{V(t) + \lambda V(t)dt | x(t), r(t) = k_1, r'(t) = k_2, t\}$$

$$= E\{e^{\lambda t}V(x(t), t, \tau(t))dt | x(t), r(t) = k_1, r'(t) = k_2, t\}$$

$$= E\{e^{\lambda t}V(x(t), t, r(t + dt), r'(t))\}$$

$$= e^{\lambda t}V(x(t), t, r(t), r'(t) + dt) | x(t), r(t) = k_1, r'(t) = k_2, t$$

$$+ o(dt)$$

which is in accordance with Lemma 2.2. In the following, let us complete the proof by considering the following two cases: $r(t) = r(t) = i$ and $r'(t) = i, r(t) = j$, respectively, where $i, j \in S$ and $j \neq i$.

**Case 1.** $r(t) = r(t) = i$. In this case, only the true mode switches may occur. Using the conclusion in [5], it follows

$$E\{e^{\lambda t}V(x(t), t, t, r(t) + dt, r'(t))\} | x(t), r(t) = r'(t) = i, t\}$$

$$= \sum_{j=1}^N \pi_{ij}^0 e^{\lambda t}V(x(t), t, i, j)$$

$$= \sum_{j=1}^N \pi_{ij}^0 e^{\lambda t}V(x(t), t, j, i)dt$$

and

$$E\{e^{\lambda t}V(x(t), t, r(t), r(t) + dt) | x(t), r(t) = r'(t) = i, t\}$$

$$= \pi_{i}^0 [e^{\lambda t}V(x(t), t, i, i) - e^{\lambda t}V(x(t), t, i, i)dt]$$

Then,

$$E\{V(t + dt) | x(t), r(t) = r'(t) = i, t\}$$

$$= E\{V(t) | x(t), r(t) = r'(t) = i, t\}$$

$$+ [\lambda e^{\lambda t}V(x(t), t, i, i) + e^{\lambda t}L V(x(t), t, i, i)]dt + o(dt)$$

where $L$ is defined in (6). Taking the expectation on the both sides of (11), then, we have

$$D^T E\{e^{\lambda t}V(x(t), t, i, i)\} = E\{\lambda e^{\lambda t}V(x(t), t, i, i)$$

$$+ e^{\lambda t}L V(x(t), t, i, i)\}$$

**Case 2.** $r'(t) = i, r(t) = j$. This situation corresponds to the detection delay, and it is assumed that the true mode $r(t)$ doesn't switch during this short time lapse. The only possible switch is that $r'(t)$ switches from $i$ to $j$, corresponding to the end of the transient, and this switch occurs on the average after $\frac{1}{\pi_{ij}}$ seconds. Then,
\[ E\left[ e^{2tV(x(t),t,r(t+dt),r'(t))} \bigg| x(t),r(t) = j \right] = \eta_{j}\left[ e^{2tV(x(t),t,j,j)} - e^{2tV(x(t),t,j,i)} \right] dt = 0 \]

and
\[ E\left[ e^{2tV(x(t),t,r(t+dt),r'(t))} \bigg| x(t),r(t) = j \right] = \eta_{j}\left[ e^{2tV(x(t),t,j,i)} - e^{2tV(x(t),t,j,j)} \right] dt \]

Thus, similar to (11), we can get
\[ D^{+}E\left[ e^{2t \mathcal{L}V(x(t),t,j,i)} \right] = E\left[ \lambda e^{2t \mathcal{L}V(x(t),t,j,i)} \right] \]
\[ + e^{2t \mathcal{L}V(x(t),t,j,i)} \]

where \( \lambda \) is defined in (7).

Combining (12) and (13), and considering the arbitrary of \( i \) and \( j \), it follows
\[ D^{+}E\{ V(t) \} = E\{ L.V(t) \} = \lambda E\{ V(t) \} \]
\[ + e^{2t \mu} E\{ L.V(x(t),t,r(t),r'(t)) \} \]

for all \( t \geq 0 \). Thus, we complete the proof. \( \Box \)

Using Lemma 3.1, the criteria of the \( p \)th moment exponentially stability for system (3) is obtained.

**Theorem 3.1**: If there exist \( c_{1} > 0 \), \( c_{2} > 0 \), \( \mu > 0 \), \( 0 < \zeta < 1 \), and \( V(x(t),t) \in \mathbb{C}^{2,1} n \times n + S \times S \times ? \), such that
\[ c_{1} |x(t)|^{p} \leq V(x(t),t,F(t)) \leq c_{2} |x(t)|^{p} \]

For any \( l \in \mathbb{R}_{+} \), there exist \( \lambda_{1} > 0 \) and \( \lambda_{2} \geq 0 \) such that
\[ L.V(x(t),t,F(t)) \]
\[ \leq \begin{cases} -\lambda_{1} V(x(t),t,F(t)), & t \in [\tau_{l-1}, \tau_{l}] \\ \lambda_{2} V(x(t),t,F(t)), & t \in [\tau_{l-1}, \tau_{l}] \end{cases} \]

Moreover,
\[ V(x(\tilde{t}),\tilde{t},F(\tilde{t})) \leq \mu V(x(\tilde{t}),\tilde{t},F(\tilde{t})) \]

and there also exist some \( \tilde{t} \) in \( [0,\lambda_{1}) \) and \( \tilde{t} \) in \( (\lambda_{2},\infty) \) such that
\[ \mu^{2} e^{(\lambda_{1}+\lambda_{2})d} - \mu \leq \zeta^{2} \]

Then, system (3) is \( p \)th moment exponentially stable.

**Proof**: According to (9) in Lemma 3.1, we have
\[ D^{+}E\{ V(x(t),t,F(t)) \} = E\{ L.V(x(t),t,F(t)) \} \]

for any \( t \in [\tau_{l-1}, \tau_{l}] \cup [\tau_{l-1}, \tau_{l}] \), \( l \in \mathbb{R}_{+} \), with \( \tau_{l} = 0 = \tau_{0} = t_{0} = 0 \).

On the one hand, from (14) and (15), one can obtain
\[ E\{ V(x(t_{0},t_{0},0,0) \} \leq c_{2} E\{ |x_{0}|^{p} \} \]

and
\[ E\{ V(x(t),t_{0},0,0) \} \leq c_{2} E\{ |x_{0}|^{p} \} e^{-\tilde{t}(t-0)} \]

for \( t \in [0,\tilde{t}) \). Combining the continuity and (16), we have
\[ E\{ V(x(\tilde{t}),t_{0},0,0) \} \leq \mu E\{ V(x(\tilde{t}),t_{0},F(\tilde{t})) \} \leq \mu c_{2} E\{ |x_{0}|^{p} \} e^{-\tilde{t}(t-0)} \]

Let \( U(t,F(t)) = e^{\tilde{t}V(x(t),t,F(t))} \). We claim that for any \( t \in [\tau_{2l-1}, \tau_{2l+1}] \),
\[ E\{ U(t,F(t)) \} \leq \mu E\{ U(\tau_{2l-1},F(\tau_{2l-1})) \} e^{(\tilde{t}_{2l}+\tilde{t}_{2l})d} \]

We will complete the proof by considering the following three cases: \( t \in [\tau_{2l-1}, \tau_{2l}] \), \( t = \tau_{2l} \) and \( t \in (\tau_{2l}, \tau_{2l+1}) \).

First, when \( t \in [\tau_{2l-1}, \tau_{2l}] \), from (15), it is easy to get
\[ E\{ U(t,F(t)) \} \leq \mu E\{ U(\tau_{2l-1},F(\tau_{2l-1})) \} e^{(\tilde{t}_{2l}+\tilde{t}_{2l})d} \]

By considering the continuity at time \( t = \tau_{2l} \), it has
\[ E\{ U(t,F(t)) \} \leq \mu E\{ U(\tau_{2l-1},F(\tau_{2l-1})) \} e^{(\tilde{t}_{2l}+\tilde{t}_{2l})d} \]

Finally, when \( t \in (\tau_{2l}, \tau_{2l+1}) \), it has
\[ E\{ U(t,F(t)) \} \leq -\mu E\{ U(t,F(t)) \} \leq 0 \]

Clearly, we have
\[ E\{ U(t,F(t)) \} \leq E\{ U(\tau_{2l},F(\tau_{2l})) \} \]
which means (21) holds on \( (\tau_{2l}, \tau_{2l+1}) \). And further, it holds on \( [\tau_{2l-1}, \tau_{2l+1}] \). Thus,
\[ E\{ V(x(t),t,F(t)) \} \leq \mu E\{ V(x(\tau_{2l-1}),F(\tau_{2l-1})) \} e^{(\tilde{t}_{2l}+\tilde{t}_{2l})d} \]
\[ x e^{(\tilde{t}_{2l}+\tilde{t}_{2l})d} \]

i.e.,
\[ E\{ V(x(t_{l+1}),t_{l+1},F(t_{l+1})) \} \leq \mu E\{ V(x(t_{l},t_{l},F(t_{l})) \} e^{(\tilde{t}_{2l+1}-\tilde{t}_{2l})d} \]

For any \( t \geq \tilde{t} = t_{1} \), iterating (23) from \( l = 1 \) to \( l = N_{t}(t_{1}+1) \), one can obtain
\[ E\{ V(x(t_{l},t_{l},F(t_{l})) \} \leq \mu^{2} E\{ V(x(t_{l}(N_{t}(t_{1}+1)),t_{l}(N_{t}(t_{1}+1)),F(t_{l}(N_{t}(t_{1}+1)))) \} x e^{(\tilde{t}_{2l}+\tilde{t}_{2l})d} \]
\[ \leq \mu^{2} E\{ V(x(t_{l}(N_{t}(t_{1}+1)),t_{l}(N_{t}(t_{1}+1)),\tilde{t}_{2l})d} \]
\[ x e^{(\tilde{t}_{2l}+\tilde{t}_{2l})d} \]
\[ \leq \mu^{2} E\{ V(x(t_{l}(N_{t}(t_{1}+1)),t_{l}(N_{t}(t_{1}+1)),\tilde{t}_{2l}+\tilde{t}_{2l}) \} \}
\[ x e^{(\tilde{t}_{2l}+\tilde{t}_{2l})d} \]

for \( t \geq \tilde{t} = t_{1} \). Combining the continuity and (16), we have
\[ E\{ V(x(\tilde{t}),t_{2},F(t_{2})) \} \leq \mu E\{ V(x(\tilde{t}),t_{2},F(\tilde{t})) \} \]
\[ \leq \mu c_{2} E\{ |x_{0}|^{p} \} e^{-\tilde{t}(t-0)} \]


\[ \leq E\{e^{2(N_{1}(t)\omega_{1}^{-1})}e^{(N_{1}(t)\omega_{1}^{-1})}(\bar{\lambda}_{1}+\bar{\lambda}_{2})d}\mu e^{(\bar{\lambda}_{1}+\bar{\lambda}_{2})d} \]

\times E\{V(x(t),t,F(t))\}e^{-\bar{\lambda}_{1}(t-\tau(t))} \]

\[ = E\{e^{2N_{1}(t)\omega_{1}^{-1}}e^{N_{1}(t)\omega_{1}}(\bar{\lambda}_{1}+\bar{\lambda}_{2})d}\} \]

\times E\{V(x(t),t,F(t))\}e^{-\bar{\lambda}_{1}(t-\tau(t))} \]

Combining (20) with (25), we arrive at

\[ E\{V(x(t),t,F(t))\} \leq E\{e^{2N_{1}(t)\omega_{1}^{-1}}e^{N_{1}(t)\omega_{1}}(\bar{\lambda}_{1}+\bar{\lambda}_{2})d}\} \]

\times \min_{x_{0}}|x_{0}|^{p} \] for any \( t \geq t_{0} \).

Based on Lemma 2.1, let \( s = 2\ln(\mu) + (\bar{\lambda}_{1} + \bar{\lambda}_{2})d \) in (8), there exists a positive number \( M > 0 \) such that

\[ e^{-\bar{\lambda}_{1}d}E\{e^{2N_{1}(t)\omega_{1}^{-1}}e^{N_{1}(t)\omega_{1}}(\bar{\lambda}_{1}+\bar{\lambda}_{2})d\} \]

\[ \leq Me^{-\bar{\lambda}_{1}d + e^{\mu s}e^{(\bar{\lambda}_{1}+\bar{\lambda}_{2})d}d - \bar{\lambda}_{1}d} \]

Then, \( E\{V(x(t),t,F(t))\} \leq Me^{-\bar{\lambda}_{1}(t-\tau(t))}e^{\mu s}e^{(\bar{\lambda}_{1}+\bar{\lambda}_{2})d}d \}

\[ \leq M + 1 < \infty \].

Then, for any \( M + 1 < \infty \), then, from (14), we have

\[ E\{\|x(t)\|^{p}\} \leq e^{-\bar{\lambda}_{1}(t-\tau(t))}e^{\mu s}e^{(\bar{\lambda}_{1}+\bar{\lambda}_{2})d}d \}

Thus, we complete the proof.

**Remark 3.1:** As usual, \( \bar{\lambda}_{1} \) and \( \bar{\lambda}_{2} \) denote the minimal stability margin and maximal instability margin, respectively.

**Remark 3.2:** From hypothesis (17), for fixed \( \bar{\lambda}_{1}, \mu \) and \( \zeta \), a larger instability margin \( \bar{\lambda}_{2} \) or a larger upper bound on detection delay \( d \) can be compensated by a smaller \( \zeta \). In other words, the \( p \)th moment exponentially stability of system (3) can be guaranteed by a sufficiently small mode transition rate.

The following two corollary can be get directly from the Theorem 3.1 and its proof. Its proof is omitted here.

**Corollary 3.1:** When \( d = 0 \), i.e., the switches between the mode-dependent controller and the system are synchronous, then if \( \mu < \frac{\bar{\lambda}_{1} + \bar{\lambda}_{2}}{\pi} \), and the conditions (9)-(11), hold, system (3) under a strictly synchronous controller \( u(t) \) is \( p \)th moment exponentially stable.

### 4. APPLICATION AND EXAMPLE

In the section, using the conclusion in Theorem 3.1, the asynchronous stability of a special class of nonlinear systems with Markovian jumping parameters is considered. A numerical example with simulation results is given to demonstrate the effectiveness of the proposed methods.

Consider the following asynchronous system

\[ \xi(t) = A\{r(t)\}x(t) + B\{r(t)\}u(t) + f(t,x(t),r(t)) \]

And we assume \( |f(t,x(t),r(t))| \leq \|u(r(t))\| \|x(t)\| \) . We consider the following mode-dependent controller

\[ u(t) = K(t)u(d(t)) \]

For convenience, when \( r(t) = i \), for any operate \( h \), let \( h_{i} \) denotes \( h(i) \). Then, the closed-loop system is

\[ \xi(t) = (A_{i} + B_{j}K_{i})x(t) + f_{j}(t,x(t)) \]

In the section, using the conclusion in Theorem 3.1, for any \( \epsilon > 0 \), when \( FF^{T} \leq I \) . Then, for any time-interval \( [\tau_{2j-1}, \tau_{2j}] \), we have

\[ \xi(t) = (A_{i} + B_{j}K_{i})x(t) + f_{j}(t,x(t)) \]

\[ \leq \xi^{T}(t)\Psi_{i,j}\xi(t) \]

where \( \Psi_{i,j} \) is given to demonstrate the effectiveness of the proposed methods.

Consider the following asynchronous system

\[ \xi(t) = A\{r(t)\}x(t) + B\{r(t)\}u(t) + f(t,x(t),r(t)) \]

and the hypothesis (17). Moreover, \( \{P_{mn}\}_{m,n \in S} \) satisfies (16). Thus system (28) is 2th moment exponentially stable.

Let \( X_{i} \) and \( \tilde{X}_{i} \) denote \( P_{ii}^{-1} \) and \( P_{i}^{-1} \), respectively. Using \( X_{i} \) pre- and post- multiply the left terms of matrix inequality (31), and using Schurs complement lemma, (31) is equivalent to
where

\[
\Pi_{11} = X_i A_i^T + A_i X_i + e_2 I + \pi_i \nu_i + \lambda_4 X_i
\]

\[
+ 2B_j K_j X_i, \quad \Pi_{22} = -\frac{1}{\sigma(j)} X_{ij}, \quad \Pi_{33} = -\frac{1}{\sigma(j)} X_{ij},
\]

\[
\Pi_{44} = -\frac{1}{\sigma(j)} X_{ij}, \quad \Pi_{55} = -\frac{1}{\sigma(j)} X_{ij}
\]

and

\[
\Pi_{66} = -\epsilon_2 \frac{1}{\| U_j \|^2} I.
\]

Similarly, using \( X_{ij} \) pre- and post- multiply the left terms of matrix inequality (32), and using Schurs complement lemma again, then (32) is equivalent to

\[
\begin{bmatrix}
\Pi_{11} & X_{ij} & X_{ij} \\
* & -\frac{1}{\sigma_{ji}} X_{ij} & 0 \\
* & * & -\epsilon_1 \frac{1}{\| U_j \|^2} I
\end{bmatrix} \leq 0
\]

where

\[
\Pi_{11} = X_{ij} A_i^T + A_i X_{ij} + 2B_j K_j X_{ij} + e_2 I - \sigma_{ji} X_{ij} - \lambda_2 X_{ij}
\]

Then, we give the following numerical example.

**Example 4.1:** To demonstrate the effectiveness, we choose the parameters in system (30) as

\[
A_1 = \begin{bmatrix}
0.2 & 0.3 \\
0.1 & 0.2
\end{bmatrix}, \quad B_1 = \begin{bmatrix}
-0.3 & 0 \\
0.1 & -0.2
\end{bmatrix}, \quad f_1(t, x(t)) = \begin{bmatrix}
0.1 \cos(t) & 0.1 \\
0 & -0.1 \sin(t)
\end{bmatrix} x(t)
\]

\[
f_2(t, x(t)) = \begin{bmatrix}
0.1 \cos(t)^2 & 0 \\
0 & 0.1 \sin(t)
\end{bmatrix} x(t)
\]

Then, we have

\[
| f_1(t, x(t)) | \leq \| U_1 \| \| x(t) \|, \quad | f_2(t, x(t)) | \leq \| U_2 \| \| x(t) \|,
\]

where

\[
U_1 = \begin{bmatrix}
0.1 & 0.1 \\
0 & -0.1
\end{bmatrix}, \quad U_2 = \begin{bmatrix}
0.1 & 0 \\
0 & 0.1
\end{bmatrix}
\]

We also assume that \( d = 0.5 \), and

\[
\Pi = \begin{bmatrix}
-0.1 & 0.1 \\
0.1 & -0.1
\end{bmatrix}, \quad \Pi^0 = \begin{bmatrix}
-10 & 10 \\
10 & -10
\end{bmatrix}
\]

According to above analysis, we choose \( \zeta = 0.99 \), \( \mu = 1.01 \), \( \epsilon_1 = 0.034 \), \( \epsilon_2 = 0.5 \), \( \lambda_1 = 0.5 \), \( \lambda_2 = 0.5 \), \( \lambda_2 = 0.5 \), and

\[
P_{11} = P_{22} = \begin{bmatrix}
3.0246 & 0.0000 \\
0.0000 & 3.0246
\end{bmatrix}, \quad P_{12} = P_{21} = \begin{bmatrix}
3.0231 & 0.0000 \\
0.0000 & 3.0231
\end{bmatrix}, \quad K_1 = \begin{bmatrix}
7.1791 & 0.5000 \\
4.3395 & 11.0186
\end{bmatrix}, \quad K_2 = \begin{bmatrix}
20.9837 & 11.2418 \\
0.5000 & 10.2418
\end{bmatrix}
\]

then, all the conditions in Theorem 3.1 are satisfied. Thus, the closed-loop system (30) with the above parameters is 2th moment exponentially stable.

Fig.1. Switching signal \( r(t) \) and the detected \( \hat{r}(t) \).

Fig.2. Response curve of \( x(t) \) under \( u(t) = 0 \).

The simulation results are shown in Fig.1-Fig.3. Among them, Fig.1 shows the Markovian switching signal which includes the real switching signal and the detected switching signal. Fig.2 shows the state
trajectories under control input $u(t) = 0$. Obviously, system (28) under $u(t) = 0$ is unstable. Fig.3 shows the response curves of system (28) under control input $u(t)$ in (29). It can be seen that the closed-loop system (30) is stable.

![Fig.3. Response curve of $x(t)$ under the control input in (29).](image)

5. CONCLUSION

In this paper, the problems of stability for a class of Markovian jumping nonlinear systems with time-delay in detection of switching signal is investigated. In the model, the detection delay is modeled as a random type with the Markovian theory described, and dependent on the actual Markovian switching signal. By considering the statistical properties of Markov process, and trying to construct the constraint relationships between instability margin or detection delay and the mode transition rate, the sufficient criterion for pth moment exponentially stability is derived. It is shown that the stability of the considerable systems can be guaranteed by a sufficient small mode transition rate. The main results are also extended to a special class of Markovian jumping nonlinear systems, and a numerical example with simulation results is given to demonstrate the effectiveness.

REFERENCES


