Networked predictive control of Hammerstein systems

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Abstract: This article mainly deals with the control and stability problems of networked Hammerstein with nonlinear input. A novel predictive controller design method is proposed to offset the effect of network delay and data dropout. The controller gain which depends on the time delay of the feedback channel is time-variant. Since we assume that the state is not measurable, the control signal is based on the state estimated by the observer. As for the nonlinear part of the input, we assume it satisfies a sector constraint and treat it as a input inaccuracy. Theoretical results are presented for the closed-loop stability by modeling the system as time-delay Hammerstein system with nonlinear inputs. A second-order Hammerstein system is implemented to show the enhanced performance of this control method.

Key Words: Networked control system, Hammerstein, nonlinear, delay, dropout.

1 Introduction

Networked Control Systems (NCSs) are a class of systems in which the links among sensors and controllers as well as the links among controllers and actuators are connected via networks. In recent years, with the development of networks, Internet has already spreaded every corner of the world which allows a cluster of devices to be linked together. Systems controlled through networks is increasingly popular as well as those practical applications, such as high-speed paper production machine, power generation plants and petrochemical processing facilities.

NCSs have many appealing strengths, such as reduced-cost, simple-installation, tele-controlled and easy to maintain. A demand for smarter sensors is increasing. Since we have further expectations on the networks reliability of tele-control. We can also benefit from the way networks transmitting data for a set of data are packed and sent together once a time. ([1]-[4]) The application of networks makes it possible to control a large scale distributed systems with a powerful central controller.

However, networked control systems have lots of constraints as well. The network-induced delays and data dropouts degraded the performance of the systems and even make the systems unstable. Due to data quantization and bandwidth limits, the network control systems is hard to implement.

Many related works have done about networked control problems. The works mainly deal with the compensation methods of the network-induced delay and strategies based on various methodologies such as gain-scheduling, control and communication codesign, robust control techniques has been suggested to design the predictive controller for linear time-invariant systems. The queuing method [11], the optimal stochastic control method [12], the sampling time scheduling method [13], and the hybrid system stability analysis method [15]. However, a fixed controller gain was used in those works. It would lead to a more conservative conclusion. And a new approach need to be developed to address this problem.

Fig. 1: The networked control system model

Zhao has proposed a two-step predictive controller to deal with the static nonlinear input of the Hammerstein systems in his paper [1]. It assumed the state can all be obtained by the sensor. Unfortunately, the states of real systems in many cases are impossible to be detected. It is necessary to design an observer to track the states of systems. In this paper we calculate the control sequence based on the states estimated by the observer and the stability issue is complemented using switch system theory. Xia at [3] designed a controller based on the estimated states, but the controller gain is fixed which results in a more conservative conclusion. We improve it by design various gains based on different backward delays that can greatly enhance the performance and make the results less conservative.

The reminder of the paper is organized as follows. In section 1, the problem formalization and the model of the system to be controlled is proposed. In section 2, the predictive controller design method is given, which consists of a state observer a matrix selector. We also analyzed the stability issue in this section. Section 3 shows the effectiveness of this method by a second-order system, we can see that it really enhance the performance of the system. The conclusion is given in section 4.
2 Design of NPCs with delays and dropouts in both forward and backward channels

Consider the following Hammerstein system $S$ with nonlinear input:

$$
S : \begin{cases} 
    x(k+1) = Ax(k) + Bv(k) \\
    y(k) = Cx(k) \\
    v(k) = f(u(k))
\end{cases} \quad (1a) \quad (1b) \quad (1c)
$$

Where $x(k) \in \mathbb{R}^m, U(k) \in \mathbb{R}^m, y(k) \in \mathbb{R}^l$ are the system state, input and output respectively. $v(k) \in \mathbb{R}^m$ is the intermediate input. And $k = 1, 2, 3, \ldots$ is a nonlinear function. $A \in \mathbb{R}^{m \times m}, B \in \mathbb{R}^{m \times n}, C \in \mathbb{R}^{l \times n}$ are the system matrices.

In this section the controller to compute the intermediate input $v(k)$ is designed and $u(k)$ is calculated by $f^{-1},$ which we do not need the function $f$ to be reversible. We can calculate it in various ways.

Some assumptions are made:

A1) $(A, B)$ is completely controllable and $(A, C)$ is completely observable.

A2) the number of data packet delay and dropout in the forward and backward channel is bounded by $M$ and $N$ respectively.

A3) all the nodes in the system is time synchronized and each data packet $\Delta u(k|k - \tau_{sc})$ sent from the controller to the actuator and from the sensor to the controller is stamped with a time label to notify its age.

A3) the control horizon is larger then the maximum data delay and dropout.

A4) the applied inputs and backward delay is known to the controller.

A5) the states of the system are not all measurable, so a state observer is needed to estimate the states.

Remark1: The internal clocks of the network control nodes are synchronized using the clock synchronization algorithm given in Real-time control systems with delays. First one node of the system is selected as master, the others are slaves that set their clock askew relative to the master. Each slave sends their master a time-stamped data packet, upon receiving the data packet, the master stamped and echo it back to the slaves. Then least-square fitting technology is adopted to the transmit time, echo time and arrival time. The fitted line is then used by the slaves to recalibrate the clock so that it askew relative to the master. In the presence, the resynchronization is done every 10s without degrade the performance of the system.

Remark2: The feedback and forward delay depends on the network condition. For simplicity reasons, an assumption is made that the delay and dropout has a maximum upper bound $M$ and $N$. And they are randomized between 1 and the maximum bound. The package received on the controller and actuator side may be disordered. Hence buffers which are able to sequence the packages with time-stamp are set at both the sides of controller and actuator. And only the latest package will be used by the controller and the actuator. The states used by the controller at time $k$ is:

$$
x(k) = \min_{\tau_{sc}, k^*} \{ X(k - \tau_{sc}, k^*) \in \Gamma_k \} \quad (2)
$$

The states used by the actuator at time $k$ is:

$$
u(k) = \min_{\tau_{sc}, k^*} \{ U(k - \tau_{sc}, k^*) \in \Upsilon_k \} \quad (3)
$$

where $\Gamma_k$ is the set of state sequence available at time $k$. $\Upsilon_k$ is the set of control sequence available at time $k$.

3 Designing the predictive controller and handling the nonlinearity of the input

3.1 Designing the predictive controller based on LGPC

Firstly, the optimal control sequence by the linear generalized predictive control algorithm is designed. Then the nonlinear part is complemented by the $f$ o $f^{-1}$.

Let $M$ denotes the upper bound of data packet delay and dropout in the forward channel and $N$ in the feedback channel.

A predictive controller determines a sequence of optimal control sequence sent to the actuator. The actuator then choose the input signal in the latest packet.

The objective function of the system is:

$$
J_{k, \tau_{sc}} = \tilde{X}(k|k - \tau_{sc})Q\tilde{X}(k|k - \tau_{sc}) + \Delta V^T(k|k - \tau_{sc})R\Delta V'(k|k - \tau_{sc}) \quad (4)
$$

Where

$$
\tilde{X}(k|k - \tau_{sc}) = [\hat{x}(k+1|k - \tau_{sc})]^T \hat{x}(k+2|k - \tau_{sc})^T \ldots \hat{x}(k+N_u|k - \tau_{sc})^T \quad (5)
$$

and

$$
\Delta V'(k|k - \tau_{sc}) = [\Delta v(k|k - \tau_{sc})|k - \tau_{sc}) \ldots \Delta v(k+N_u|k - \tau_{sc})]^T \quad (6)
$$

where $J_{k, \tau_{sc}}$ is the objective function at time $k$. $\tau_{sc}$ is the corresponding delay in the backward channel. $\tilde{X}(k|k - \tau_{sc})$ is the states estimated by the observer at $k$ from $k + 1$ to $k + N_p$, $\Delta V'(k|k - \tau_{sc})$ is the optimal predicted control sequence at $k$. Our objective is to minimize $J_{k, \tau_{sc}}$. As the calculated input is $\Delta V'(k|k - \tau_{sc})$. We rewrite the system function as follow:

$$
S \left\{ \begin{array}{l}
    \hat{x}(k+1) = \hat{A}\hat{x}(k) + \hat{B}\hat{v}(k) \\
    \hat{y}(k) = \hat{C}\hat{x}(k)
\end{array} \right. \quad (7a) \quad (7b)
$$

Where

$$
\hat{x}(k) = [x(k)^T \ v(k-1)]^T \\
\Delta \hat{v}(k) = \hat{v}(k) - \hat{v}(k - 1)
$$

$$
\hat{A} = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix}, \hat{B} = \begin{bmatrix} B \\ I \end{bmatrix}, \hat{C} = [C \ 0]
$$

Since we assume the states can not be measured directly from the output. Here we adopt the observer to estimate the states up to $k + N_p$, where $N_p$ is the prediction upper bound.

$$
\hat{x}(k+1|k) = \hat{A}\hat{x}(k|k - 1) + \hat{B}\hat{v}(k) + L(y(k) - \hat{C}\hat{x}(k|k - 1)) \quad (8)
$$
\[ \dot{x}(k - \tau_{sc} + 1|k - \tau_{sc}) = \dot{A}\dot{x}(k - \tau_{sc}|k - \tau_{sc} - 1) + \dot{B}\Delta v(k - \tau_{sc}) + L(y(k - \tau_{sc}) - \tilde{C}\hat{x}(k - \tau_{sc}|k - \tau_{sc} - 1)) \]
\[ = (\dot{A} - \dot{L}\tilde{C})\dot{x}(k - \tau_{sc}|k - \tau_{sc} - 1) + \dot{B}\Delta v(k - \tau_{sc}) + \dot{L}\tilde{C}\hat{x}(k - \tau_{sc}) \]
\[ (9) \]
\[ \dot{x}(k - \tau_{sc} + 2|k - \tau_{sc}) = \dot{A}\dot{x}(k - \tau_{sc} + 1|k - \tau_{sc}) + \dot{B}\Delta v(k - \tau_{sc} + 1|k - \tau_{sc}) \]
\[ (10) \]
\[ \ldots \]
Which results in:
\[ \dot{x}(k+N_p|k-\tau_{sc}) = A^{N_p+\tau_{sc}-1}(\dot{A} - \dot{L}\tilde{C})\dot{x}(k-\tau_{sc}|k-\tau_{sc}-1) \]
\[ + A^{N_p+\tau_{sc}-1}\dot{L}\tilde{C}\hat{x}(k-\tau_{sc}) \]
\[ + \sum_{j=\tau_{sc}}^{N_p-1-j} A^{N_p-1-j}B\Delta v(k+j|k-\tau_{sc}) \]
\[ j = 1, 2, 3 \ldots (11) \]
Let
\[ \hat{X} = [\dot{x}(k+1|k-\tau_{sc})^T \quad \dot{x}(k+2|k-\tau_{sc})^T \quad \ldots \quad \dot{x}(k+N_p|k-\tau_{sc})^T]^T \]
Then
\[ \hat{X}(k|k-\tau_{sc}) = E_{\tau_{sc}} \hat{x}(k-\tau_{sc}) + F_{\tau_{sc}} \Delta v'(k|k-\tau_{sc}) + G_{\tau_{sc}} \hat{x}(k-\tau_{sc}) \]
\[ (12) \]
Where
\[ E_{\tau_{sc}} = [A^{\tau_{sc}} L\tilde{C} \quad A^{\tau_{sc}+1} L\tilde{C} \ldots A^{\tau_{sc}+N_p-1} L\tilde{C}]^T \]
\[ (F_{\tau_{sc}})^T = (A^{\tau_{sc}+i-1} B, j, i \leq \tau_{sc}) \]
\[ G_{\tau_{sc}} = [A^{\tau_{sc}} (A - L\tilde{C}) \quad A^{\tau_{sc}+1} (A - L\tilde{C}) \ldots A^{\tau_{sc}+N_p-1} (A - L\tilde{C})]^T \]
\[ (13) \]
Note that \( E_{\tau_{sc}}, F_{\tau_{sc}} \) and \( G_{\tau_{sc}} \) varies with backward delay \( \tau_{sc} \). Then the optimal \( \Delta v(k|k-\tau_{sc}) \) can be obtained by minimize \( \partial J/\partial \Delta v^* \).

We get that:
\[ \Delta V(k|k-\tau_{sc}) = -M_{\tau_{sc}}(F_{\tau_{sc}}^T Q F_{\tau_{sc}} + R)^{-1} F_{\tau_{sc}}^T Q \]
\[ \dot{x}(k-\tau_{sc}) + G_{\tau_{sc}} \hat{x}(k-\tau_{sc})|k-\tau_{sc} - 1)) \]
\[ \]
Where
\[ \Delta V(k|k-\tau_{sc}) = [\Delta v(k+1|k-\tau_{sc}) \quad \Delta v(k+2|k-\tau_{sc}) \quad \ldots \quad \Delta v(k+N_p|k-\tau_{sc})]^T \]
\[ M_{\tau_{sc}} = [0_{M_{N_k} \times Mk} \quad I_{M_{N_k} \times Mk}] \]
\[ (14) \]
As we will prove in section 4, the observer will be convergent at the state \( x \). We can take the state feedback gain \( K_{\tau_{sc}} \) as:
\[ K_{\tau_{sc}} = -M_{\tau_{sc}}(F_{\tau_{sc}}^T Q F_{\tau_{sc}} + R)^{-1} F_{\tau_{sc}}^T Q (E_{\tau_{sc}} + G_{\tau_{sc}}) \]
\[ (15) \]
Remark 3: The \( E_{\tau_{sc}}, F_{\tau_{sc}}, K_{\tau_{sc}} \) differs from various feedback delay. If we calculate a \( K_{\tau_{sc}} \) every time a data packet arrives at the controller. It is a great computation burden on the controller. However we notice that there is a limit of the backward delay \( N \) as we have made a assumption before. Then we have a matrix select on the controller side. We store all the possible matrix in it. Namely \( \mathcal{F} = [E_1 \ E_2 \ldots E_N], \mathcal{G} = [G_1 \ G_2 \ldots G_N], \mathcal{M} = [M_1 \ M_2 \ldots M_N] \). Then every time a data packet arrives. We chose the corresponding \( K_{\tau_{sc}} \) to calculate the optimal input sequence \( \Delta V(k|k-\tau_{sc}) \). And \( \Delta U(k|k-\tau_{sc}) \) is computed based on \( \Delta V(k|k-\tau_{sc}) \) and \( f^{-1} \). Since the optimal input is based on the states, however the state is not available, A state observer is used to estimate the state \( \hat{x}(k|k-\tau_{sc}) \).

### 3.2 Complementing the nonlinear part of the input

Note that \( f \) is a nonlinear function with \( \Delta u(k) \) as input, we calculate \( \Delta v(k) \) based on the algorithm mentioned above. When we compute \( \Delta u(k) \) we need to know the inverse of \( f \). We need not require that \( f \) is invertible. We can obtain \( f^{-1} \) in many other ways.

\[ \Delta u(k|k-\tau_{sc}) = f^{-1}(\Delta v(k|k-\tau_{sc})) \]
\[ (16) \]
\[ \Delta v(k|k-\tau_{sc}) \] is the real input, \( \Delta v(k|k-\tau_{sc}) \) is the intermediate input. Notice that the input for the origin system should be \( u(k|k-\tau_{sc}) \), and

\[ u(k|k-\tau_{sc}) = f^{-1}(\Delta v(k|k-\tau_{sc})) \]
\[ (17) \]
Taking the nonlinearity of the system into consideration \( f(\Delta u(k|k-\tau_{sc})) \neq f(\Delta v(k|k-\tau_{sc})) - f(\Delta v(k-1|k-1-\tau_{sc})) \)

Thus the compensation of \( f^{-1} \) do not offset the effect of the nonlinear completely. This inaccuracy introduce a nonlinear disturbance that appears in the form of \( f \circ f^{-1} \). \( f \circ f^{-1} \ neq 1 \) in general. It makes reasonable to assume that \( f \circ f^{-1} \) meet some sector constraints. As we assumed in the following assumption.

A6) the nonlinearity of the function \( f \) meets the sector constraint that \( \varepsilon \leq f \circ f^{-1} \leq \bar{\varepsilon} \). Then there exits a \( \varepsilon_k \) satisfies \( \underline{\varepsilon} \leq \varepsilon_k \leq \bar{\varepsilon} \).

\[ w(k|k-\tau_{sc}) = f \circ f^{-1}(K_{\tau_{sc}} f(\hat{x}(k|k-\tau_{sc}))) \]
\[ = \varepsilon_k(K_{\tau_{sc}} f(\hat{x}(k|k-\tau_{sc}))) \]
\[ (18) \]
Then the system can be rewritten as:

\[ S : \begin{cases} \dot{x}(k+1) = \hat{A}\hat{x}(k) + \hat{B}w(k) \\ y(k) = \hat{C}\hat{x}(k) \end{cases} \]
\[ (19a) \]
\[ (19b) \]

### 3.3 Summarizing the approach above

Now the steps of the predictive controller designing is detailed as follow:

S1) the sensor detects the state and the output of the plant, labeled it with a time stamp and send it to the controller side.

S2) the controller read the time stamp of the packet from the plant side, select the corresponding \( K_{\tau_{sc}} \) from the matrix selector based on the time delay of this data packet. The
intermediate input $\Delta v(k|k-\tau_{sc})$ is calculated. Then optimal sequence $\Delta v(k|k-\tau_{sc})$ is computed through $V$ and $f^k(1)$. 
S3) the optimal control sequence $\Delta u(k|k-\tau_{sc})$ is packed with a time stamp and send to the buffer of the plant side.
S4) during a time interval there may be more than one data packet arrive at the buffer. Only the latest data is store. Others are abandoned. The plant would chose the ith data of the buffer and send it to the plant.
S5) return to S1.

3.4 Closed-loop system
In order to analysis the stability of the controlled system. We need to get the states of the plant and observer at this moment. We predict the output of the system based on the state up to $k-\tau_{sc}$, where $\tau_{sc}$ is the backward delay.

\[ \hat{x}(k+1|k) = A\hat{x}(k|k) + B\Delta v(k) + L(y(k)-C\hat{x}(k|k-1)) \] 

(20)

\[ \hat{x}(k-\tau_{sc} + 1|k-\tau_{sc}) = A\hat{x}(k-\tau_{sc}|k-\tau_{sc}-1) + B\Delta v(k-\tau_{sc}) 
+ L(y(k-\tau_{sc}) - C\hat{x}(k-\tau_{sc}|k-\tau_{sc}-1)) 
= (A - L\hat{C})\hat{x}(k-\tau_{sc}|k-\tau_{sc}-1) + B\Delta v(k-\tau_{sc}) + L\hat{C}\hat{x}(k-\tau_{sc}) 
\hat{x}(k-\tau_{sc} + 2|k-\tau_{sc}) = (A - L\hat{C})\hat{x}(k-\tau_{sc} + 1|k-\tau_{sc}) 
+ B\Delta v(k-\tau_{sc} + 1) \] 

(21)

Which results in:

\[ \hat{x}(k|k-\tau_{sc}) = A^{\tau_{sc}}(\hat{A} - L\hat{C})\hat{x}(k-\tau_{sc}|k-\tau_{sc}-1) + A^{\tau_{sc}-1}L\hat{C}\hat{x}(k-\tau_{sc}|k-\tau_{sc}) 
+ \sum_{j=1}^{\tau_{sc}} A^{\tau_{sc}-j}B\Delta v(k-\tau_{sc} + j) \] 

(22)

In most cases, there exists a forward delay $\tau_{ca}$, and we need to predict the state at time $k + \tau_{ca}$.

\[ \hat{x}(k+1|k-\tau_{sc}) = A\hat{x}(k|k-\tau_{sc}) + B\Delta v(k) \] 

(23)

\[ \hat{x}(k+2|k-\tau_{sc}) = A\hat{x}(k+1|k-\tau_{sc}) + B\Delta v(k+1) \] 

(24)

\[ \hat{x}(k+\tau_{ca}|k-\tau_{sc}) = (A + BK_{\tau_{ca}})\hat{x}(k-\tau_{sc} - 1) 
+ A^{\tau_{ca}}(\hat{A} - L\hat{C})\hat{x}(k-\tau_{sc}|k-\tau_{sc}-1) 
+ A^{\tau_{ca}-1}L\hat{C}\hat{x}(k-\tau_{sc}) 
+ \sum_{j=1}^{\tau_{ca}} A^{\tau_{ca}-j}B\Delta v(k-\tau_{sc} + j) \] 

(25)

Thus the closed-loop system is:

\[ \hat{x}(k+1) = A\hat{x}(k) + BK_{\tau_{ca}}(A + BK_{\tau_{ca}})^{\tau_{ca}}(\hat{A} - L\hat{C}) \hat{x}(k-\tau_{sc} - 1) 
+ A^{\tau_{ca}}\hat{x}(k-\tau_{sc}) 
+ \sum_{j=1}^{\tau_{ca}} A^{\tau_{ca}-j}B\Delta v(k-\tau_{sc} + j) \] 

(26)

\[ \hat{x}(k+1) = A\hat{x}(k+1) + BK_{\tau_{ca}}(A + BK_{\tau_{ca}})^{\tau_{ca}}(\hat{A} - L\hat{C}) \hat{x}(k-\tau_{sc} - 1) 
+ A^{\tau_{ca}}\hat{x}(k-\tau_{sc}) 
+ \sum_{j=1}^{\tau_{ca}} A^{\tau_{ca}-j}B\Delta v(k-\tau_{sc} + j) \] 

(27)

3.5 Augmented system
Let's $X(k) = \{\hat{x}(k)^T \hat{x}(k+1)^T \cdots \hat{x}(k+M+N)^T\}$ where $w(k+1) \ 0 \ w(k+2) \ \ldots \ w(k+M+N) \ \hat{x}(k-1)^T \hat{x}(k-2)^T \cdots \hat{x}(k-M-N)^T \hat{x}(k-N)^T$ 

Then the augmented system is

\[ X(k+1) = \Lambda(\tau_{sc}, \tau_{ca})X(k) \] 

(28)

Where

\[ \Lambda(\tau_{sc}, \tau_{ca}) = \begin{bmatrix} \Lambda_{11}(\tau_{sc}, \tau_{ca}) & \Lambda_{12}(\tau_{sc}, \tau_{ca}) & \Lambda_{13}(\tau_{sc}, \tau_{ca}) \\ \Lambda_{21}(\tau_{sc}, \tau_{ca}) & \Lambda_{22}(\tau_{sc}, \tau_{ca}) & \Lambda_{23}(\tau_{sc}, \tau_{ca}) \\ \Lambda_{31}(\tau_{sc}, \tau_{ca}) & \Lambda_{32}(\tau_{sc}, \tau_{ca}) & \Lambda_{33}(\tau_{sc}, \tau_{ca}) \end{bmatrix} \] 

(29)

Let

\[ M_{sc} = K_{\tau_{sc}}(A + BK)^{\tau_{ca}}(\hat{A} - L\hat{C}) \] 

\[ M_{sc-1} = K_{\tau_{sc}}(A + BK)^{\tau_{ca}-1} \hat{B} \] 

\[ M_{ca-\tau_{ca}} = K_{\tau_{ca}}(A + BK)^{\tau_{ca}-1}(\hat{A} - L\hat{C}) \] 

Then the augmented system is

\[ X(k+1) = \Lambda(\tau_{sc}, \tau_{ca})X(k) \] 

(30)

\[ A_{11}(\tau_{sc}, \tau_{ca}) = \begin{bmatrix} \hat{A} & 0 & \ldots & 0 & BM_{\text{ie}} & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 & (M+N)_{n} & 0 & \ldots & 0 \end{bmatrix} \] 

(31)

\[ A_{12}(\tau_{sc}, \tau_{ca}) = \begin{bmatrix} BM_{0} & BM_{1} & \ldots & BM_{\tau_{ca}-1} & 0 & \ldots & 0 \end{bmatrix} \] 

(32)

\[ A_{13}(\tau_{sc}, \tau_{ca}) = \begin{bmatrix} 0 & \ldots & 0 & BM_{\text{ie}} & 0 & \ldots & 0 \end{bmatrix} \] 

(33)

\[ A_{21}(\tau_{sc}, \tau_{ca}) = \begin{bmatrix} 0 & \ldots & 0 & M_{\text{ie}} & 0 & \ldots & 0 \end{bmatrix} \] 

(34)

\[ A_{22}(\tau_{sc}, \tau_{ca}) = \begin{bmatrix} 0 & \ldots & 0 & M_{\text{ie}} & 0 & \ldots & 0 \end{bmatrix} \] 

(35)

\[ A_{23}(\tau_{sc}, \tau_{ca}) = \begin{bmatrix} 0 & \ldots & 0 & M_{\text{ie}} & 0 & \ldots & 0 \end{bmatrix} \] 

(36)

\[ A_{31}(\tau_{sc}, \tau_{ca}) = \begin{bmatrix} L\hat{C} & 0 & \ldots & 0 & barBM_{\text{ie}} & 0 & \ldots & 0 \end{bmatrix} \] 

(37)

\[ A_{32}(\tau_{sc}, \tau_{ca}) = \begin{bmatrix} BM_{0} & BM_{1} & \ldots & BM_{\tau_{ca}-1} & 0 & \ldots & 0 \end{bmatrix} \] 

(38)
3.6 Stability theorem

Theorem 1 If there exists a positive definite matrix

\[ P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \]

such that the following inequality holds

\[ \begin{bmatrix} \Lambda(\tau_{sc}, \tau_{ca}) & 0 \\ 0 & \Lambda(\tau_{sc}, \tau_{ca}) \end{bmatrix}^T \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} \Lambda(\tau_{sc}, \tau_{ca}) & 0 \\ 0 & \Lambda(\tau_{sc}, \tau_{ca}) \end{bmatrix} \begin{bmatrix} P_1 \\ 0 \\ P_2 \\ 0 \end{bmatrix} < 0 \] (39)

Then the system is stable and the observer can be used to estimate the state used in the predictive controller.

Proof 1 Let

\[ V(k) = \begin{bmatrix} X(k) \\ \hat{x}(k) - \bar{x}(k) \end{bmatrix}^T \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} X(k) \\ \hat{x}(k) - \bar{x}(k) \end{bmatrix} \]

be the Lyapunov candidate for the augmented system (28) and the observer. Then the incremental \( V(k) \) for the system can be obtained from:

\[ \Delta V(k) = V(k + 1) - V(k) \]

\[ = \begin{bmatrix} X(k + 1) \\ \hat{x}(k + 1) - \bar{x}(k + 1) \end{bmatrix}^T \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} X(k + 1) \\ \hat{x}(k + 1) - \bar{x}(k + 1) \end{bmatrix} \]

\[ - \begin{bmatrix} X(k) \\ \hat{x}(k) - \bar{x}(k) \end{bmatrix}^T \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} X(k) \\ \hat{x}(k) - \bar{x}(k) \end{bmatrix} \]

\[ = \begin{bmatrix} X(k) \\ \hat{x}(k) - \bar{x}(k) \end{bmatrix}^T \begin{bmatrix} \Lambda(\tau_{sc}, \tau_{ca}) & 0 \\ 0 & \Lambda - \bar{L}\bar{C} \end{bmatrix} \begin{bmatrix} P_1 \\ 0 \\ P_2 \end{bmatrix} \begin{bmatrix} \Lambda(\tau_{sc}, \tau_{ca}) & 0 \\ 0 & \Lambda - \bar{L}\bar{C} \end{bmatrix} \begin{bmatrix} X(k) \\ \hat{x}(k) - \bar{x}(k) \end{bmatrix} \]

\[ = \begin{bmatrix} X(k) \\ \hat{x}(k) - \bar{x}(k) \end{bmatrix}^T \begin{bmatrix} \Lambda(\tau_{sc}, \tau_{ca}) & 0 \\ 0 & \Lambda - \bar{L}\bar{C} \end{bmatrix} \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} \Lambda(\tau_{sc}, \tau_{ca}) & 0 \\ 0 & \Lambda - \bar{L}\bar{C} \end{bmatrix} \begin{bmatrix} X(k) \\ \hat{x}(k) - \bar{x}(k) \end{bmatrix} \]

\[ < 0 \] (40)

From Theorem 1, we know that

\[ \begin{bmatrix} \Lambda(\tau_{sc}, \tau_{ca}) & 0 \\ 0 & \Lambda - \bar{L}\bar{C} \end{bmatrix}^T \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} \Lambda(\tau_{sc}, \tau_{ca}) & 0 \\ 0 & \Lambda - \bar{L}\bar{C} \end{bmatrix} \begin{bmatrix} P_1 \\ 0 \\ P_2 \\ 0 \end{bmatrix} < 0 \] (41)

Thus

\[ \Delta V(k) < 0 \]

P1 ensures the stability of the closed system. P2 guarantees that the observer can track the system states correctly. Thus the feasibility of P lead to the stability of the novel method proposed in this paper.

4 Simulation

To verify the effectiveness of the approach proposed. A second-order Hammerstein system is presented here. Firstly, we control the system with the novel approach in this paper. Second, we apply the conventional linear quadratic optimal control method to this system.

\[ A = \begin{bmatrix} 0.5 & 0.7 \\ 0 & 1 \end{bmatrix} B = \begin{bmatrix} 0.08 \\ 0.3 \end{bmatrix} C = [1 \ 0] \]

The observer poles are placed in [0.1 0.1]. Using the Matlab, we get that the observer gain is \( L = [2.68 - 4.86] \). The gain of the conventional method is \( K = [-0.07 \ 0.073 \ 0.24] \). And the gain of the novel approach is a vector that has 10 elements, 10 is the control length. Then the control signal varies with different backward delay, every control increment signal is calculated based on the immediate delay. We can get the state trajectory as in the picture follows.

![Fig. 2: The comparison between conventional and novel method: second-order system](image)

From the figure we can see that this novel method has smaller overshoot and marginally faster steady time. Though we do not see obvious improvement, the result is very meaningful. Since the states can not be get directly, we must design an observer. And the time delay is stochastic, We can not predict the next delay, the control signal applied on the system is not available, too. Taking all those into considerations, we have made a big progress.

5 Conclusions

The controller designing and stability analysis of the networked control system with nonlinear input is studied. As the state may not be obtained directly from the system, An observer is adopted to estimate and predict the state by iteration. The controller gain is time-variant based on different time-delays. Notice that the feedback gain used in the observer is time-invariant which is different from the \( K_{\tau_{sc}} \) we design in the controller. At the actuator side, an buffer will order the sequence arrived and select the appropriate input signal. A numerical examples has been showed to illustrate the effectiveness of this approach. As for the stability, an augmented system is designed, the state of the origin system
is included in the augmented system, then the stability of the augmented system would result in the stability of the origin system.

In the industry applications of the networked control system, the plant itself has inertia, which would take the form of state delay in the system function. This problem should be studied in the near future.

References


1. Networked predictive control of Hammerstein systems

Accession number: 20144300119247
Authors: Wang, Tian-Tian (1); Kang, Yu (1); Zhao, Yun-Bo (2)
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Issue date: September 11, 2014
Publication year: 2014
Article number: 6895945
Language: English
ISSN: 19341768
E-ISSN: 21612927

Document type: Conference article (CA)
Conference name: Proceedings of the 33rd Chinese Control Conference, CCC 2014
Conference date: July 28, 2014 - July 30, 2014
Conference location: Nanjing, China
Conference code: 108070

Sponsor: Systems Engineering Society of China; Technical Committee on Control Theory of Chinese Association of Automation
Publisher: IEEE Computer Society

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Number of references: 15

Main heading: Networked control systems

Controlled terms: Controllers - Delay control systems - Feedback - Nonlinear systems - Time delay
Uncontrolled terms: Closed loop stability - delay - dropout - Hammerstein - Hammerstein system - Networked predictive controls - nonlinear - Predictive controller

Classification code: 713 Electronic Circuits - 731 Automatic Control Principles and Applications - 731.1 Control Systems - 732.1 Control Equipment
DOI: 10.1109/ChiCC.2014.6895945

Compendex references: YES

Database: Compendex
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Data Provider: Engineering Village
Record 1 of 1

Title: Networked predictive control of Hammerstein systems

Author(s): Wang, TT (Wang, Tian-tian); Kang, Y (Kang, Yu); Zhao, YB (Zhao, Yun-Bo)

Book Group Author(s): IEEE


Times Cited in Web of Science Core Collection: 0

Total Times Cited: 0

Usage Count (Last 180 days): 0

Usage Count (Since 2013): 0

Cited Reference Count: 15

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Accession Number: WOS:000366482805178

Language: English

Document Type: Proceedings Paper

Conference Title: 33rd Chinese Control Conference (CCC)

Conference Date: JUL 28-30, 2014

Conference Location: Nanjing, PEOPLES R CHINA

Author Keywords: Networked control system; Hammerstein; nonlinear; delay; dropout

KeyWords Plus: INTEGRATED COMMUNICATION; FEEDBACK CHANNELS; COMPENSATION; ALGORITHM; STABILITY; DELAYS

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Publisher: IEEE

Publisher Address: 345 E 47TH ST, NEW YORK, NY 10017 USA

Web of Science Categories: Automation & Control Systems

Research Areas: Automation & Control Systems

IDS Number: BE900D

ISSN: 1948-9439

29-char Source Abbrev.: CHIN CONT DECIS CONF

Source Item Page Count: 6